

Analysis Test 1: 2016–17

1. If A is a set of real numbers, what conditions on A are required for $\sup(A)$ to exist as a real number? Explain carefully in words what $\sup(A)$ means. **(3)**
2. Write down the Archimedean property of the real numbers. **(2)**
3. You are given that $\sqrt{7} = 2.645731$ to seven significant figures. Is it true that there is an irrational number q for which

$$2.64573 < q < \sqrt{7} \quad ?$$

Give reasons for your answer. **(3)**

4. Write down the precise definition of what it means for a sequence (a_n) of real numbers to converge to a limit l , using ϵ and N . **(2)**
5. Make a guess as to the value of $\lim_{n \rightarrow \infty} \left(\frac{5}{7} - \frac{3}{2n^{1/3}} \right)$, and use the Archimedean property to prove that this is correct. **(9)**
6. Carefully explain why $\lim_{n \rightarrow \infty} \frac{1 - \sin(n)}{n^2}$ exists, and find its value. (You may use without proof the fact that $\lim_{n \rightarrow \infty} k/n^2 = 0$, for any $k \in \mathbb{R}$.) **(6)**