

Analysis Test 2: 2016–17

1. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function with domain D_f .
 - (a) Give the precise definition, using sequences, of what it means for f to have a limit $l \in \mathbb{R}$ at the point $a \in D_f$.
 - (b) Give an equivalent formulation of (a) using ϵ and δ .
 - (c) What does it mean, in terms of limits, for f to be continuous at a ?
 - (d) What does it mean, in terms of limits, for f to be differentiable at a ? **(9)**

2. Let $f(x) = \frac{(x+6)}{(36-x^2)(1-x)}$

- (a) Write down the domain of f .
- (b) Do either of $\lim_{x \rightarrow 6} f(x)$ or $\lim_{x \rightarrow -6} f(x)$ exist? Calculate each limit explicitly, or explain why divergence occurs. **(10)**

3. Consider the function

$$f(x) = \begin{cases} 3-x & \text{if } x < 0 \\ 1+5x^2 & \text{if } x \geq 0. \end{cases}$$

Is f continuous at $x = 0$? Is f differentiable at $x = 0$? Give explicit calculations, or quote a relevant theorem, to support your conclusions. **(6)**

[You may draw a picture to help with intuition, but there are no marks for this.]

4. Consider the function

$$f(x) = \begin{cases} 1-x & \text{if } x < 0 \\ 1+x & \text{if } x \geq 0. \end{cases}$$

Is f continuous at $x = 0$? Is f differentiable at $x = 0$? Give explicit calculations, or quote a relevant theorem, to support your conclusions. **(10)**

[You may draw a picture to help with intuition, but there are no marks for this.]