

MAS221 Analysis (Semester 1)– Solutions to Problems 20 to 24.

20. Limit is 1. (a) (i) 30, (ii) 300, (iii) 3000, (iv) 30000, (v) 30000000000.
(b) Given any $\epsilon > 0$, by the Archimedean property of \mathbb{R} , we can find $N \in \mathbb{N}$ so that $N > 3/\epsilon$. Then for all $n > N$, we have

$$|a_n - 1| = 3/n < 3/N < \epsilon.$$

21. Limits are (a) 1, (b) 0, (c) 0, (d) 0. Argue as in the solution to 20)b) but this time use the Archimedean property to find $N \in \mathbb{N}$ which is greater than: (a) $\frac{1}{\epsilon}$, (b) $\frac{3}{\epsilon}$, (c) $\frac{1}{\sqrt{\epsilon}}$, (d) $\frac{1}{\epsilon^2}$.
22. n th term is $\frac{n}{2^{n-1}}$. Given $\epsilon > 0$, use the Archimedean property to find $N \in \mathbb{N}$ so that $N > \frac{1+2\epsilon}{4\epsilon}$. Then straightforward algebra shows that for $n > N$ we get $|\frac{n}{2^{n-1}} - \frac{1}{2}| < \epsilon$, as required.
23. Since (x_n) converges to x , given any $\epsilon > 0$, there exists $N \in \mathbb{N}$ so that if $n > N$ then $|x_n - x| < \epsilon$. But then by Theorem 1.3.1,

$$||x_n - |x|| \leq |x_n - x| < \epsilon,$$

and the result follows. Converse is false in general, - e.g consider $(-1)^n$. However, the converse is true in the case where (x_n) is a null sequence (see Problem 30), i.e. $x = 0$. For in that case, for all $n \in \mathbb{N}$,

$$||x_n| - 0| = |x_n| = |x_n - 0|.$$

24. Given any $\epsilon > 0$ we can find $N \in \mathbb{N}$ so that if $n > N$ then

$$|b_n - 0| = b_n \leq a_n = |a_n - 0| < \epsilon.$$