MAS331/6352 METRIC SPACES 2019–19

Assignment 1

1. Let $I = [0, \frac{\pi}{2}]$, $f(x) = \sin x$, and $g(x) = \cos x$. In $C(I)$, find $d_1(f, g)$ and $d_\infty(f, g)$.

2. Let $a_1, \ldots, a_n$ be strictly positive numbers (i.e. $a_i > 0$ for $1 \leq i \leq n$). Show that $(\mathbb{R}^n, d)$ is a metric space where

$$d(\mathbf{x}, \mathbf{y}) = \sum_{i=1}^{n} a_i |x_i - y_i|,$$

   for each $\mathbf{x} = (x_1, \ldots, x_n), \mathbf{y} = (y_1, \ldots, y_n)$.

3. Chapter 1, Problem 3(b,c,e).

4. Let (X, d) be an arbitrary metric space. Use the triangle inequality to show that

$$|d(x, z) - d(y, z)| \leq d(x, y)$$

   for all $x, y, z \in X$, and hence deduce that

$$|d(x, y) - d(a, b)| \leq d(x, a) + d(y, b)$$

   for all $x, y, a, b \in X$.

Assignment given out on Tuesday 9th October, for handing in on Tuesday 16th October.