# Ornstein-Uhlenbeck Processes with Jumps in Hilbert Space

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# 1 Stochastic Evolution Equations

General set-up:

$H$ is a real separable Hilbert space. Inner product $\langle \cdot, \cdot \rangle$.
$\mathcal{L}(H)$ - bounded linear operators on $H$.

$Y = (Y(t), t \geq 0)$ is an $H$-valued process satisfying

$$dY(t) = [JY(t-) + B(Y(t-))]dt + C(Y(t-))dX(t), \qquad (1.1)$$

- $B$ and $C$ are suitable (Lipshitz) mappings $H \to \mathcal{L}(H)$;
- $X$ is an $H$-valued semimartingale;
- $J$ is the infinitesimal generator of a one-parameter semigroup $(S(t), t \geq 0)$ on $H$.

Motivation: SPDES driven by *space-time white noise* can be reformulated as SEEs driven by $L^2$(space)-valued noise.

Applications - e.g. Burgers turbulence, interest rate models.

$X$ = Brownian motion (da Prato and Zabzcyk).

We take

- $X$ is a Lévy process
- $B = 0, C(\cdot) = C \in \mathcal{L}(H)$

We study the infinite dimensional *Langevin equation*:

$$dY(t) = JY(t) + CdX(t).$$

## 2 Lévy Processes in $H$

Filtered space $(\Omega, \mathcal{F}, (\mathcal{F}_t, t \geq 0), P)$.

$X = (X(t), t \geq 0)$ is a Lévy process, i.e.

- stationary and independent increments;
- $X(0) = 0$ (a.s.)
- càdlàg paths, stochastic continuity.

*Lévy-Khintchine formula* (Varadhan)

$$\mathbb{E}(e^{i\langle u, X(t)\rangle}) = e^{-t\eta(u)},$$

for all $t \geq 0, u \in H$.

$$\begin{aligned} \eta(u) &= -i\langle b, u\rangle + \frac{1}{2}\langle u, Qu\rangle \\ &+ \int_{H-\{0\}} [1 - e^{i\langle y, u\rangle} + i\langle y, u\rangle 1_{\hat{B}}(y)]\nu(dy) \end{aligned} \qquad (2.2)$$

Characteristics $(b, Q, \nu)$:

- $b \in H$,
- $Q$ is a positive, self-adjoint, trace class operator on $H$,
- $\nu$ is a *Lévy measure* on $H - \{0\}$, i.e.

$$\int_{H-\{0\}} (||y||^2 \wedge 1)\nu(dy) < \infty.$$

$$(\hat{B} = \{x \in H; 0 < |x| < 1\}).$$

*Poisson random measure* on $\mathbb{R}^+ \times (H - \{0\})$:

$$N(t, A) = \#\{0 \leq s \leq t; \Delta X(s) \in A\}.$$

*Compensator* $\tilde{N}(dt, dx) = N(dt, dx) - dt\nu(dx)$.

*Lévy-Itô Decomposition*

$$X(t) = bt + B_Q(t) + \int_{||x||<1} x\tilde{N}(t, dx) + \int_{||x||\geq 1} xN(t, dx).$$

$B_Q$ is a *Brownian motion* with covariance operator $Q$:

$$\mathbb{E}(\langle u, B_Q(s)\rangle\langle v, B_Q(t)\rangle) = (s \wedge t)\langle u, Qv\rangle.$$

Poisson analogue:

$$\mathbb{E}\left(\left\langle u, \int_A x\tilde{N}(s, dx)\right\rangle \left\langle v, \int_B x\tilde{N}(t, dx)\right\rangle\right) = (s\wedge t)\int_{A\cap B} \langle u, T_x v\rangle\nu(dx),$$

where $T_x v := \langle x, v\rangle x$.

$T_x = |x\rangle\langle x|$ in Dirac's "bra-ket" notation.)

To see this observe that

$$\begin{aligned} & \mathbb{E}\left(\left\langle u, \int_A x\tilde{N}(s, dx)\right\rangle \left\langle v, \int_B x\tilde{N}(t, dx)\right\rangle\right) \\ = \ & \mathbb{E}\left(\int_A \langle u, x\rangle\tilde{N}(s, dx) \int_B \langle v, x\rangle\tilde{N}(t, dx)\right) \\ = \ & (s \wedge t)\int_{A\cap B} \langle u, x\rangle\langle v, x\rangle\nu(dx). \end{aligned}$$

$T_x$ is easily seen to be positive, self-adjoint and trace-class.

## 3 Stochastic Integration

Aim : To define $\int_0^T F(s)dB(s) + \int_0^T \int_{\hat{B}} F(s,x)x\tilde{N}(ds,dx)$,

Integrators: - *martingale valued measure*:

$$M((s,t],A) = (B_Q(t) - B_Q(s))\delta_0(A) + \int_s^t \int_{A-\{0\}} x\tilde{N}(dt,dx).$$

Covariance field $R_x = \begin{cases} Q \text{ if } x = 0 \\ T_x \text{ if } x \neq 0 \end{cases}$

Integrands: $\mathcal{H}_2(T)$ is real Hilbert space of all (predictable) $F : \Omega \times [0,T] \times \hat{B} \rightarrow \mathcal{L}(H)$ for which

$$||F||_2^2 := \mathbb{E}\left(\int_O^T \int_{\hat{B}} \text{tr}(F(s,x)R_x F(s,x)^*)\nu(dx)ds\right) < \infty.$$

Begin with step-functions of the form

$$F = \sum_{i=0}^{N_1} \sum_{j=0}^{N_2} F_{ij} 1_{(t_i,t_{i+1}]} 1_{A_j},$$

with each $F_{ij}$ being $\mathcal{F}_{t_i}$-measurable.

$$\text{Define } I_T(F) = \sum_{i=0}^{N_1} \sum_{j=0}^{N_2} F_{ij} M((t_i,t_{j+1}],A_j).$$

$$\text{Compute } \ \mathbb{E}(||I_T(F)||^2) = ||F||_2^2,$$

and extend by density in usual way.

For illustration - review Brownian motion case (da Prato and Zabzcyk).

$$F = \sum_{i=0}^{n} F_i 1_{(t_i,t_{i+1}]}.$$

Use two facts from Hilbert space theory. Let $(e_n, n \in \mathbb{N})$ be an orthonormal basis:-

(i) (Parseval's formula) If $\psi \in H$

$$||\psi||^2 = \sum_{n=0}^{\infty} |\langle \psi, e_n \rangle|^2.$$

(ii) If $T$ is a trace-class operator

$$\mathrm{tr}(T) = \sum_{n=0}^{\infty} \langle e_n, Te_n \rangle.$$

$$\begin{aligned}
\mathbb{E}(||I_T(F)||^2) &= \sum_{j=1}^{n}\sum_{k=1}^{n} \mathbb{E}(\langle F_j(B(t_{j+1}) - B(t_j)), F_k(B(t_{k+1}) - B(t_k))\rangle) \\
&= \sum_{j=1}^{n} \mathbb{E}(||F_j(B(t_{j+1}) - B(t_j))||^2) \\
&= \sum_{j=1}^{n}\sum_{m=0}^{\infty} \mathbb{E}(|\langle F_j(B(t_{j+1}) - B(t_j)), e_m\rangle|^2) \quad \text{by (i)} \\
&= \sum_{j=1}^{n}(t_{j+1} - t_j) \sum_{m=1}^{\infty} \langle F_j^* e_m, QF_j^* e_m \rangle \\
&= \sum_{j=1}^{n}(t_{j+1} - t_j)\mathrm{tr}(F_j Q F_j^*) \\
&= ||F||_2^2.
\end{aligned}$$

In general, the condition $||F||_2^2 < \infty$ can be rewritten as

$$\mathbb{E}\left(\int_0^T \int_{\hat{B}} ||F(t,x)T_x^{\frac{1}{2}}||_{HS}\nu(dx)dt\right) < \infty,$$

where $||\cdot||_{HS}$ is the Hilbert-Schmidt norm, i.e. $||C||_{HS} = \mathrm{tr}(CC^*)$ for $C \in \mathcal{L}(H)$. The set of all $C \in \mathcal{L}(H)$ for which $||C||_{HS} < \infty$ is a Hilbert space with respect to the inner product $(C, D)_{HS} = \mathrm{tr}(CD^*)$, which we denote as $\mathcal{L}_2(H)$.

$\mathcal{L}_2(H)$ is a two-sided $L(H)$-ideal with $||C_1DC_2||_{HS} \leq ||C_1||.||C_2||.||D||_{HS}$, for all $C_1, C_2 \in \mathcal{L}(H), D \in \mathcal{L}_2(H)$. From this we easily deduce that

$$\int_0^T \int_{\hat{B}} \mathbb{E}(||F(t,x)||^2)\mathrm{tr}(T_x)\nu(dx)dt < \infty \tag{3.3}$$

is a sufficient condition for existence of stochastic integrals.

# 4 Wiener-Lévy Integrals

Deterministic case:

$F : \mathbb{R}^+ \to \mathcal{L}(H)$ measurable and locally square-integrable.

Using the Lévy Itô decomposition, we can define

$$\begin{aligned} \int_0^t F(s)dX(s) &:= \int_0^t F(s)bds \quad \text{(Bochner integral)} \\ &+ \int_0^t F(s)dB_Q(s) + \int_0^t \int_{||x||<1} F(s)x\tilde{N}(ds,dx) \\ &+ \int_0^t \int_{||x||\geq 1} F(s)xN(ds,dx) := \sum_{0\leq s\leq t} F(s)\Delta X(s)1_{\hat{B}^c}(\Delta X(s)) \end{aligned}$$

In this case (3.3) is always satisfied since the LHS becomes

$$\int_0^T ||F(s)||^2 ds \int_{\hat{B}} \mathrm{tr}(T_x)\nu(dx) \quad \text{and,}$$

$$\mathrm{tr}(T_x) = \sum_{n\in\mathbb{N}} |\langle e_n, x\rangle|^2 = ||x||^2.$$

Important example: If $(S(t), t \geq 0)$ is a $C_0$-semigroup then there exists $M \geq 1, \beta \geq 0$ such that for all $t \geq 0$,

$$||S(t)|| \leq Me^{\beta t}. \tag{4.4}$$

so the *stochastic convolution* $\int_0^t S(t-s)dX(s)$ exists.

*Properties of Wiener-Lévy Integrals*

**Proposition 4.1** *For each $t \geq 0, \int_0^t F(s)dX(s)$ is infinitely divisible and its characteristic exponent is given by*

$$\lambda_{t,F}(u) := \int_0^t \eta(F(s)^* u)ds, \tag{4.5}$$

*for each $u \in H$.*

*Proof.* The integral on the right hand side exists since there exists $K > 0$ such that for all $s \geq 0, u \in H$,

$$\begin{aligned} ||\eta(F(s)^* u)||^2 &\leq K(1 + ||F(s)^* u||^2) \\ &\leq K(1 + ||F(s)||^2 ||u||^2). \end{aligned}$$

For each $u \in H$, we define the complex valued process $(M_u(t), t \geq 0)$ by

$$M_u(t) = \exp\left\{ i \left\langle u, \int_0^t F(s)dX(s) \right\rangle \right\},$$

for each $t \geq 0$. Using Itô's formula, we obtain

$$\begin{aligned} M_u(t) &= -\int_0^t M_u(s-)\eta(F(s)^* u)ds + i \int_0^t M_u(s-)\langle F(s)^* u, dB(s)\rangle \\ &+ \int_0^t \int_{H-\{0\}} M_u(s-)(e^{i\langle F(s)^* u, x\rangle} - 1)\tilde{N}(ds, dx). \end{aligned}$$

After taking expectations, we find that

$$\mathbb{E}\left( \exp\left\{ i \left\langle u, \int_0^t F(s)dX(s) \right\rangle \right\} \right) = \exp\left\{ -\int_0^t \eta(F(s)^* u)ds \right\},$$

as was required.

To see that the stochastic integral is infinitely divisible, first note that for each $n \in \mathbb{N}, \frac{\eta}{n}$ is Sazonov continuous, hermitian, negative definite and vanishing at zero, hence there exists a càdlàg Lévy process $(X_n(t), t \geq 0)$ such that for each $u \in H, t \geq 0$,

$$\mathbb{E}(e^{i\langle u, X_n(t)\rangle}) = e^{-t\frac{\eta(u)}{n}}.$$

Hence

$$\left[\mathbb{E}\left(\exp\left\{i\left\langle u, \int_0^t F(s)dX(s)\right\rangle\right\}\right)\right]^{\frac{1}{n}} = \mathbb{E}(e^{i\langle u, \int_0^t F(s)dX_n(s)\rangle}),$$

and the result follows. □

**Corollary 4.1** *For each $t \geq 0$, $\int_0^t F(s)dX(s)$ has characteristics $(b_t, Q_t, \nu_t)$, where*

$$b_t := \int_0^t F(s)bds + \int_0^t \int_{H-\{0\}} F(s)x[1_{\hat{B}}(x) - 1_{\hat{B}}(F(s)x)]\nu(dx)ds,$$

$$Q_t := \int_0^t F(s)QF(s)^* ds,$$

$$\nu_t(A) := \int_0^t \nu(F(s)^{-1}A)ds, \text{ for each } A \in \mathcal{B}(H - \{0\}).$$

Define $I_F(t) = \int_0^t F(t)dX(t)$.

Other useful properties:

- $(I_F(t), t \geq 0)$ is an additive process.
- The laws $(p_{I_F}(t), 0 \leq t \leq T)$ are tight.
- If $t \to ||F(t)||$ is locally bounded then $t \to I_F(t)$ is stochastically continuous.

# 5 Ornstein-Uhlenbeck Process

$$dY(t) = JY(t) + dX(t), \quad Y(0) = Y_0 \ \text{ a.s.} \tag{5.6}$$

The *Ornstein-Uhlenbeck process*

$$Y(t) = S(t)Y_0 + \int_0^t S(t-s)dX(s) \tag{5.7}$$

(5.7) is the unique weak solution to (5.6), i.e. for all $u \in H$,

$$\langle u, Y(t) - Y_0 \rangle = \langle u, X(t) \rangle + \int_0^t \langle J^* u, Y(s) \rangle ds.$$

(*Chojnowska-Michalik, DA*)

$Y = (Y(t), t \geq 0)$ is a Markov process.

It induces a generalised *Mehler semigroup* $(\mathcal{T}_t, t \geq 0)$ on $C_b(H)$:

$$\begin{aligned} (\mathcal{T}_t f)(y) &= \mathbb{E}(f(Y(t))|Y_0 = y) \\ &= \int_H f(S(t)y + x)\rho_t(dx) \end{aligned} \tag{5.8}$$

where $\rho_t$ is the law of $\int_0^t S(u)dX(u)$.

Conversely linear operators defined as in (5.8) form a semigroup if and only if

$$\rho_{t+s} = \rho_t * (\rho_s \circ S(t)^{-1}) \tag{5.9}$$

(*Bogachev-Röckner-Schmuland, Fuhrman-Röckner*)

A random variable $X$ is *(operator) self-decomposable* if there exists a process $(Z(t), t \geq 0)$ independent of $X$ such that

$$X \stackrel{d}{=} S(t)X + Z(t).$$

(*c.f. Jurek, Jurek-Vervaat*)

There is a well-known connection in finite dimensions between self-decomposable distributions and stationary OU processes.

In infinite dimensions, if $Y$ is stationary OU

$$Y(0) \stackrel{d}{=} Y(t) = S(t)Y(0) + \int_0^t S(t-s)dX(s),$$

so $Y(0)$ is self-decomposable.

**Theorem 5.1** *The OU process* $(Y(t), t \geq 0)$ *is stationary iff the associated Mehler semigroup* $(\mathcal{T}_t, t \geq 0)$ *has an invariant measure* $\mu$. *In either case* $\mu$ *is the law of* $Y(t)$.

**Theorem 5.2** *If* $\mu$ *is an invariant measure for* $(\mathcal{T}_t, t \geq 0)$ *then it is self-decomposable.*

*Conversely if* $\mu$ *is self-decomposable and* $\hat{\mu}(u) \neq 0$ *for all* $u \in H$, *then there exists a Mehler semigroup with invariant measure* $\mu$.

*Proof.* $\mu$ invariant $\Rightarrow$

$$\begin{aligned} \int_H (\mathcal{T}_t f)(x)\mu(dx) &= \int_H \int_H f(S(t)x + y)\rho_t(dy)\mu(dx) \\ &= \int_H f(x)\mu(dx) \end{aligned}$$

$$\Rightarrow \mu = (\mu \circ S(t)^{-1}) * \rho_t.$$

Conversely, $\mu$ self-decomposable $\Rightarrow$

$$\begin{aligned} \mu &= (\mu \circ S(t+s)^{-1}) * \rho_{t+s} \\ &= (\mu \circ S(t)^{-1} \circ S(s)^{-1}) * (\rho_s \circ S(t)^{-1}) * \rho_t \end{aligned}$$

$$\Rightarrow \rho_{t+s} = (\rho_s \circ S(t)^{-1}) * \rho_t.$$

Argue as above to show $\mu$ invariant. □

If $(S(t), t \geq 0)$ is "stable", i.e. $\lim_{t\to\infty} S(t)u = 0$ for all $u \in H$,

- $\mu$ is unique invariant measure (hence ergodic).
- $\mu = \text{weak-}\lim_{t\to\infty} \rho_t$.
- $\mu$ is infinitely divisible (Gaussian in Brownian motion case). Its characteristics are $(b_\infty, Q_\infty, \nu_\infty)$.

From the point of view of OU processes:

$\mu$ exists iff $\int_0^\infty S(t)dX(t)$ exists in distribution.

Necessary and sufficient conditions (Chojnowska-Michalik)

(A) $\lim_{t\to\infty} \int_0^t S(t)bdx$ exists.

(B) $\int_0^\infty \text{tr}(S(t)QS(t)^*)dt < \infty$.

(C) $\int_0^\infty \int_{H-\{0\}} (||S(r)x||^2 \wedge 1)\nu(dx)dr < \infty$

(D) $\lim_{t\to\infty} \int_0^t \int_{H-\{0\}} S(r)x[1_{B_1}(S(r)(x)) - 1_{B_1}(x)]\nu(dx)ds$ exists.

Exponentially stable case: $||S(t)|| \leq Ce^{-\lambda t}$ for some $C \geq 1, \lambda > 0$ :

- (A) and (B) always hold.
- $\int_0^\infty S(t)dX(t)$ exists in distribution iff $\int_0^\infty \int_{||x||>1} S(u)xN(du, dx)$ exists in distribution.

The classical condition $\int_{||x||\geq 1} \log(1 + ||x||)\nu(dx) < \infty$ is sufficient (but not necessary) for this when $\dim(H) = \infty$.

## 6 Operator Self-Similarity

$(S(t), t \geq 0)$ is a $C_0$ semigroup with infinitesimal generator $J$. $(X(t), t \geq 0)$ is $J$-*self-similar* if for all $a > 0$:

$$X(at) \stackrel{d}{=} S(\log(a))X(t).$$

*Lamperti transformation*

Assume $(S(t), t \geq 0)$ is a group.

$$Y \text{ stationary} \Rightarrow S(\log(t))Y(\log(t)) \text{ is } J\text{-self-similar.}$$
$$X \text{ is } J\text{-self-similar} \Rightarrow S(-t)X(e^t) \text{ is stationary.}$$

(*Matache-Matache*)

# 7 The Infinitesimal Generator

$$(\mathcal{T}_t f)(x) = \int_H f(S(t)x + y)\rho_t(dy).$$

Problem: $t \to \mathcal{T}_t f$ is not continuous for the usual uniform topology $\tau_u$ on $C_b(H)$ or $UC_b(H)$.

Introduce the *mixed topology* $\tau_m$ on $C_b(H)$.

(*Goldys-Kocan, Goldys- van Neerven*)

It is locally convex and generated by the seminorms

$$\rho_{(a_n),(K_n)}(f) = \sup_{n \in \mathbb{N}} \sup_{x \in K_n} |a_n f(x)|,$$

- $(K_n, n \in \mathbb{N})$ is a sequence of compact sets in $H$.
- $a_n > 0, \lim_{n \to \infty} a_n = 0$.

$\tau_m$ is complete.
Sequential convergence: $f_n \to f$ in $\tau_m$ iff

(M1) $\sup_{n \in \mathbb{N}} \sup_{x \in H} |f_n(x)| < \infty$.

(M2) $f_n \to f$ in $\tau_{uc}$,

where $\tau_{uc}$ is topology of uniform convergence on compacta.

$$\tau_{uc} < \tau_m < \tau_u.$$

**Theorem 7.1** *$(\mathcal{T}_t, t \geq 0)$ is strongly continuous on $(C_b(H), \tau_m)$.*

*Proof.* As usual its sufficient to prove strong continuity at $t = 0$. For (M1) it's enough to consider a sequence $(t_n, n \in \mathbb{N})$ in $[0, 1]$ converging to zero. Then for $f \in C_b(H)$,

$$\sup_{n \in \mathbb{N}} \sup_{x \in H} |(\mathcal{T}_{t_n} f)(x) - f(x)| \leq 2 \sup_{x \in H} |f(x)| < \infty.$$

To establish (M2) use the fact that $\{\rho_t, t \in [0,1]\}$ is tight, hence given an arbitrary $\epsilon > 0$, there exists a compact set $L$ in $H$ such that

$$\rho_t(L) \geq 1 - \frac{\epsilon}{8||f||} \quad \text{for all} \quad t \in [0,1].$$

Fix a compact $K \subset H$. We can use uniform continuity of $f$ on compacta and strong continuity of $(S(t), t \geq 0)$ to argue that there exists $t_0 \in [0,1]$ such that $0 \leq t < t_0 \Rightarrow$

$$\sup_{x \in K} \sup_{y \in L} |f(S(t)x + y) - f(x + y)| < \frac{\epsilon}{4}.$$

Now write

$$(\mathcal{T}_t f)(x) - f(x) = I_1(f,t,x) + I_2(f,t,x),$$

where

$$I_1(f,t,x) := \int_H [f(S(t)x + y) - f(x + y)]\rho_t(dy)$$

$$\text{and} \quad I_2(f,t,x) := \int_H [f(x + y) - f(x)]\rho_t(dy).$$

Now for $0 \leq t < t_0$,

$$\begin{aligned} \sup_{x \in K} |I_1(f,t,x)| &\leq \sup_{x \in K} \int_H |f(S(t)x + y) - f(x + y)|\rho_t(dy) \\ &\leq \sup_{x \in K} \sup_{y \in L} |f(S(t)x + y) - f(x + y)| + 2||f||\rho_t(L^c) \\ &< \frac{\epsilon}{2} \end{aligned}$$

Using uniform continuity of $f$ on compacta and stochastic continuity of $Y$, it follows by a standard argument that $\sup_{x \in K} |I_2(f,t,x)| < \frac{\epsilon}{2}$, for sufficiently small $t$. Hence $\mathcal{T}_t f \xrightarrow{uc} f$ as $t \to 0$ and the result follows. □

The infinitesimal generator $\mathcal{A}$ is densely defined and closed (with respect to $\tau_m$).

$C_J^2(H) \subseteq \mathrm{Dom}(\mathcal{A})$ is dense in $(C_b(H), \tau_m)$. It comprises those $C^2$ functions $f$ whose first and second derivatives are uniformly bounded and uniformly continuous on bounded subsets of $H$ and for which

$$\mathrm{Ran}(Df) \subseteq \mathrm{Dom}(J^*) \text{ and the mapping } x \to \langle x, J^*(Df)(x)\rangle \in C_b(H),$$

where $D$ is the Fréchet derivative. On this space

$$(\mathcal{A}f)(x) = \langle x, J^*(Df)(x)\rangle + (\mathcal{L}_X f)(x),$$

where $\mathcal{L}_X$ is the infinitesimal generator of the Markov semigroup of the Lévy process $X = (X(t), t \geq 0)$:

$$\begin{aligned}(\mathcal{L}_X f)(x) &= \langle (Df)(x), b\rangle + \frac{1}{2}\mathrm{tr}((D^2 f)(x)Q) \\ &+ \int_{H-\{0\}} [f(x+y) - f(x) - \langle (Df)(x), y\rangle 1_{\hat{B}}(y)]\nu(dy).\end{aligned}$$

- $\mathcal{A}$ has a convenient core of cylinder functions.
- $\mathcal{A}$ has a pseudo-differential operator representation.
- When $b = \nu = 0$ (Gaussian case), $\mathcal{A}$ is the *Kolmogorov operator.*

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