

**Errata for “Probability on Compact Lie Groups” - by
D.Applebaum *July 2014***

On page 1, line 3 of the abstract the phrase “of ... Statistic-
stant”, should be deleted.

On p.54, +4 and p.61, +6, change \cap to \cup .

pp.179 - 201. The labels on individual appendices vanished mys-
teriously during the production process. These should be

- (A1) Basic Concepts of Topology.
- (A2) Concerning Left/Right Uniform Continuity.
- (A3) Manifolds
- (A4) The Symplectic and Spin Groups.
- (A5) Measures on Locally Compact Spaces
- (A6) Compact, Hilbert-Schmidt and Trace-Class Operators.
- (A7) Semigroups of Linear Operators.
- (A8) Cores of Closed Linear Operators on Banach Spaces.

**All of the above were rapidly corrected by Springer, so
if you bought a copy after mid-September 2014, you
need only start reading from here.**

Thanks to Uwe Franz for finding many of the following:

2, +10 Insert comma before Π^∞ , and on the next line, “field-
sand” should be “fields and”.

9, -1 The subscript on the product should be $1 \leq i, j \leq n$, and
 $\det(x)$ should be $|\det(x)|$.

12,+5 Here is a better way to describe the Lie bracket on $T_e(G)$. To each $X \in \mathfrak{g}$, associate $\tilde{X} \in \mathcal{L}(G)$ by $\tilde{X}(g) = dl_g(X)$, for all $g \in G$. Then the Lie bracket in \mathfrak{g} is defined by

$$[X, Y] := [\tilde{X}, \tilde{Y}](e).$$

17, The group Laplacian Δ defined in (1.3.8) coincides with the Laplace–Beltrami operator (i.e. $\text{div} \circ \text{grad}$) if and only if G is unimodular. This result appears in, as yet unpublished work by Ming Liao. Of course, for much of the book G is compact, hence unimodular, and so these two operators can safely be identified.

71, +14 Sugiura is misspelt.

78, -17 and -15, π should be π_λ .

79, -4 unimodular is misspelt.

80, -2 [160] should be [106].

103, + 2 Change $m/2$ to $3m/2$.

123, In Theorem 5.2.1, the convolution semigroup $(\mu_t, t \geq 0)$ cannot be assumed, in general, to satisfy $\mu_0 = \delta_e$. For a counter-example, consider normalised Haar measure m on G which is infinitely divisible, and embedded into the “constant” convolution semigroup for which $\mu_t = m$ for all $t \geq 0$.

126, -8 $C_0^{2,L}$ should be $C_0^{2,L}(G)$.

127,-5 x_n should be x_d .

132, -5 I was perhaps, a little cavalier with language in the statement of Theorem 5.3.3, in that \mathcal{L} is the infinitesimal generator of the Hunt semigroup $(P_t, t \geq 0)$ that is uniquely determined by the convolution semigroup $(\mu_t, t \geq 0)$.

183, +19 $\frac{\partial}{\partial x_j}$ should be $\frac{\partial}{\partial x_i}$.

183, +20 “to p at M ” should read “to M at p .”

188, -4 The symplectic product is not a (real) inner product. It takes values in the space Q of quaternions.

191, + 1 In the definition of *inner regularity*, A should be open.

194, +3 These orthonormal sequences in H are not necessarily complete.

194, +9 $\sigma(T)$ should be $\sigma(T) \setminus \{0\}$.

201, Reference [3] should read: “G.W.Anderson, A.Guionnet, O.Zeitouni, *An Introduction to Random Matrices*, Cambridge University Press (2010)”

208, Fourier lost his capital F in references [190] and [191].

Following a discussion with Mathew Joseph, I have a partial converse to Theorem 4.7.2 on page 112, which is very much in the spirit of Hawkes’ approach when $G = \mathbb{R}^d$:

Theorem 0.1 *If P_μ is strong Feller, then μ is absolutely continuous with respect to any right (or left)-invariant Haar measure on G .*

Proof. First assume that m is right-invariant. Suppose that $f \in B_b(G)_+$ with $m(f) = 0$. Then by Fubini’s theorem and right-invariance of m , $\int_G P_\mu f(\sigma) m(d\sigma) = \int_G \int_G f(\sigma\tau) \mu(d\tau) m(d\sigma) = 0$. Hence, $P_\mu f = 0$, (a.e.). But $P_\mu f$ is continuous by the strong Feller assumption, and so $P_\mu f = 0$. So in particular, $\mu(f) = P_\mu f(e) = 0$, and the result follows. Replace f by \tilde{f} for the case where m is left-invariant. \square