Errata for "Probability on Compact Lie Groups" - by D.Applebaum *July 2014*

On page 1, line 3 of the abstract the phrase "of ... Statistic-stant", should be deleted.

On p.54, +4 and p.61, +6, change $\cap$ to $\cup$.

pp.179 - 201. The labels on individual appendices vanished mysteriously during the production process. These should be

(A1) Basic Concepts of Topology.

(A2) Concerning Left/Right Uniform Continuity.

(A3) Manifolds

(A4) The Symplectic and Spin Groups.

(A5) Measures on Locally Compact Spaces

(A6) Compact, Hilbert-Schmidt and Trace-Class Operators.

(A7) Semigroups of Linear Operators.

(A8) Cores of Closed Linear Operators on Banach Spaces.

All of the above were rapidly corrected by Springer, so if you bought a copy after mid-September 2014, you need only start reading from here.

Thanks to Uwe Franz for finding many of the following:

2, +10 Insert comma before Π^∞, and on the next line, "field-sand" should be "fields and".

9, -1 The subscript on the product should be $1 \leqslant i, j \leqslant n$, and $\det(x)$ should be $|\det(x)|$.

12,+5 Here is a better way to describe the Lie bracket on $T_e(G)$. To each $X \in \mathfrak{g}$, associate $\widetilde{X} \in \mathcal{L}(G)$ by $\widetilde{X}(g) = dl_g(X)$, for all $g \in G$. Then the Lie bracket in $\mathfrak{g}$ is defined by

$$[X, Y] := [\widetilde{X}, \widetilde{Y}](e).$$

17, The group Laplacian Δ defined in (1.3.8) coincides with the Laplace–Beltrami operator (i.e div$\circ$grad) if and only if G is unimodular. This result appears in, as yet unpublished work by Ming Liao. Of course, for much of the book G is compact, hence unimodular, and so these two operators can safely be identified.

71, +14 Sugiura is misspelt.

78, -17 and -15, π should be π_λ.

78, -8 Replace $|||F(\lambda)|||$ with $||F(\lambda)||_{HS}$.

79, -4 unimodular is misspelt.

80, -2 [160] should be [106].

103, + 2 Change $m/2$ to $3m/2$.

123, In Theorem 5.2.1, the convolution semigroup $(\mu_t, t \geqslant 0)$ cannot be assumed, in general, to satisfy $\mu_0 = \delta_e$. For a counter–example, consider normalised Haar measure m on G which is infinitely divisible, and embedded into the "constant" convolution semigroup for which $\mu_t = m$ for all $t \geqslant 0$.

126, -8 $C_0^{2,L}$ should be $C_0^{2,L}(G)$.

127,-5 x_n should be x_d.

132, -5 I was perhaps, a little cavalier with language in the statement of Theorem 5.3.3, in that $\mathcal{L}$ is the infinitesimal generator of the Hunt semigroup $(P_t, t \geqslant 0)$ that is uniquely determined by the convolution semigroup $(\mu_t, t \geqslant 0)$.

183, +19 $\frac{\partial}{\partial x_j}$ should be $\frac{\partial}{\partial x_i}$.

183, +20 "to p at M" should read "to M at p."

188, -4 The symplectic product is not a (real) inner product. It takes values in the space Q of quaternions.

191, + 1 In the definition of *inner regularity*, A should be open.

194, +3 These orthonormal sequences in H are not necessarily complete.

194, +9 $\sigma(T)$ should be $\sigma(T) \setminus \{0\}$.

201, Reference [3] should read: "G.W.Anderson, A.Guionnet, O.Zeitouni, *An Introduction to Random Matrices*, Cambridge University Press (2010)"
208, Fourier lost his capital F in references [190] and [191].

Following a discussion with Mathew Joseph, I have a partial converse to Theorem 4.7.2 on page 112, which is very much in the spirit of Hawkes' approach when $G = \mathbb{R}^d$:

Theorem 0.1 *If P_μ is strong Feller, then μ is absolutely continuous with respect to any right (or left)-invariant Haar measure on G.*

Proof. First assume that m is right-invariant. Suppose that $f \in B_b(G)_+$ with $m(f) = 0$. Then by Fubini's theorem and right-invariance of m, $\int_G P_\mu f(\sigma) m(d\sigma) = \int_G \int_G f(\sigma\tau)\mu(d\tau)m(d\sigma) = 0$. Hence, $P_\mu f = 0$, (a.e.). But $P_\mu f$ is continuous by the strong Feller assumption, and so $P_\mu f = 0$. So in particular, $\mu(f) = P_\mu f(e) = 0$, and the result follows. Replace f by $\tilde{f}$ for the case where m is left-invariant. □