## PhD Projects

The main theme of my research is the study of random processes from both a theoretical and an applied perspective. For many years this has specifically focussed on *Lévy processes* which are named after the great 20th century probabilist Paul Lévy (1886-1971). These are essentially processes that have stationary and independent increments. On the one hand, they have a very rich structure which leads to lots of interesting mathematics. On the other hand they have recently been finding increasing applications to e.g. physics, finance, biology and engineering. One reason for this is that they form a rich class of processes which are well suited for mathematical modelling of phenomena that manifest random discontinuities.

Another major theme of my work is the study of stochastic differential equations (SDEs) and stochastic partial differential equations (SPDEs) where the evolution of a random dynamical system is described differentially through the interaction of both deterministic and random noise components. I am particularly interested in the case where the latter is a Lévy process or some variation thereof.

Here are some specific areas in which I would be interested in supervising a PhD:

1. **SPDEs**

   Recently there has been a lot of interest in studying SPDEs driven by Lévy noise where the equations are reformulated as SDEs of evolution type that are driven by a Banach space valued process. Recently Markus Riedle and I have developed a systematic approach to the use of "cylindrical Lévy processes" to drive such equations. This is a type of noise that arises naturally in applications but it is not a Banach space valued Lévy process in the usual sense.

   *Specific Project.* To investigate existence, uniqueness, path properties and ergodicity of SPDEs driven by cylindrical Lévy noise.

   *References*

   S.Peszat, J.Zabczyk, *Stochastic Partial Differential Equations with Lévy Noise*, Cambridge University Press (2007)

   D.Applebaum, M.Riedle, Cylindrical Lévy processes in Banach space, preprint (arXiv:0905.2858v1) available from `http://www.applebaum.staff.shef.ac.uk/papers.html`

2. **Stochastic Resonance**

Stochastic resonance is a phenomenon wherein small amounts of random noise can enhance the output of a system rather than degrading it. This has found a wide range of applications in physics, biology and medicine. Quite recently Patel and Kosko have shown that SDEs driven by Lévy processes can induce stochastic resonance. It would be interesting to gain a deeper understanding of how this is happening.

*Specific Project.* To find classes of Lévy processes which optimise stochastic resonance from the point of view of both speed of the transition and magnitude of the effect. Can we model stochastic resonance as a phase transition?

*References*

A.Patel and B.Kosko, Stochastic resonance in continuous and spiking neuron models with Levy noise, *IEEE Trans. Neural Netw.*, vol. 19, No. 12, pp 1993-2008, December 2008

D.Applebaum, Extending stochastic resonance for neuron models to general Levy noise, to appear in *IEEE Trans. Neural Netw.* - preprint available from `http://www.applebaum.staff.shef.ac.uk/papers.html`

3. **Probability on Groups**

Studying probability theory on groups enables us to gain insight into how "chance" and "symmetry" interact. It also leads to some beautiful mathematics.

*Specific Project 1.* Using Peter-Weyl theory, I've recently found a necessary and sufficient condition for a probability measure on a compact group to have a square-integrable density. Can this be extended to the general case where the square-integrability condition is dropped?

*Specific Project 2.* Lévy processes make good sense on Lie groups. They are Markov processes and hence induce $C_0$-semigroups which can be extended to act in $L^2(G, m)$ where $m$ is a suitable Haar measure. Under what conditions does the semigroup comprise of compact operators (for $t > 0$)? In the case where the semigroup is self-adjoint, can we give a complete description of its spectrum?

*References*

D.Applebaum, Probability measures on compact groups which have square-integrable densities, *Bulletin of the London Math Soc.*, **40**, 1038-44 (2008)

D.Applebaum, Some $L^2$ properties of semigroups of measures on Lie groups, *Semigroup Forum* **79**, 217-28 (2009)

If you want to discuss coming to Sheffield to work with me on any of these or related projects, please e-mail me on D.Applebaum@sheffield.ac.uk. For information about the Department's graduate entry procedures, go to `http://www.shef.ac.uk/pas/prospectivepg/phd_mphil/admissions.html`

Dave Applebaum, September 2009