Author's comment on "Probabilistic Trace and Poisson Summation Formulae."

In the numerical example at the end of section 5, I used the "probabilist's" Fourier transform, rather than the "analyst's" one. This matters for the calculation. So we should have $\widehat{f}(n) = e^{-2\pi|x|}$ and then

$$\sum_{n\in\mathbb{Z}} \widehat{f}(n) = 1 + \frac{2}{e^{2\pi}-1} = 1.003742,$$

which is less than the given estimate for $\sum_{n\in\mathbb{Z}} f(n)$. In fact a numerical investigation (summing 10000 terms) indicates that $\sum_{n=1}^{\infty} 1/(1+n^2)$ is approximately 1.076574, and so $\sum_{n\in\mathbb{Z}} f(n)$ is approximately 1.003678.

This constitutes convincing evidence that the Poisson summation formula does hold in this case.

In fact, we can go further and prove that it holds by using the fact (see [42], top of page 155) that the Poisson summation formula is valid when both f and $\widehat{f}$ are of moderate decrease. We only need to show this for $\widehat{f}$. Since $\lim_{|x|\to\infty}(1+|x|^2)e^{-2\pi|x|} = 0$, given any $\epsilon > 0$, there exists $R > 0$ so that if $|x| > R$, we have $(1+|x|^2)e^{-2\pi|x|} < \epsilon$. Then

$$e^{-2\pi|x|} \leq \frac{A}{1+|x|^2},$$

where $A := \max\left\{\epsilon, \sup_{|x|\leq R}(1+|x|^2)e^{-2\pi|x|}\right\}$. The same argument works for arbitrary $t > 0$, so the Poisson summation formula in that case, yields:

$$\frac{1}{\pi}\sum_{n\in\mathbb{Z}} \frac{t}{t^2+n^2} = \sum_{n\in\mathbb{Z}} e^{-2\pi t|n|} = 1 + \frac{2}{e^{2\pi t}-1}.$$

We may now conclude that the Poisson summation formula for symmetric α–stable processes on $\mathbb{R}$ holds for $\alpha = 1$ and $\alpha = 2$. The problem remains open if $1 < \alpha < 2$. The extension to rotationally invariant α–stable processes on $\mathbb{R}^d$ is straightforward.

Note also that three lines above Prop. 5.1, we are working in the case $d = 1$.