Review of "Advanced Calculus: An Introduction to Linear Analysis" by Leonard F.Richardson, J.Wiley and Sons, Inc. (2008), ISBN: 978-0-470-23288-0 £57.95 (hardback) pp. 387.

This book is based on a three semester course that is taught to undergraduates at Louisiana State University. It is aimed at readers who already have a basic understanding of one-variable calculus (at the "mathematical methods" level) and some linear algebra and who want to progress to a thoroughly rigorous course on mathematical analysis in both one and many dimensions. The novelty of the book is that the author also wants to prepare readers to eventually go further into functional analysis and he does this by integrating some preparatory material on (real-valued) sequence and function spaces and on linear operators into the book as it progresses.

The book is divided into three parts. Part I is a standard first course in analysis comprising chapters on sequences, continuity, Riemann integration, differentiation and infinite series (including Taylor's theorem) in that order. This covers all of the topics you'd expect to see in such a course but the author also introduces the Banach space $C(a, b)$ of continuous functions on $[a, b]$ at the end of the continuity chapter and proves that it is complete under the supremum norm. Similarly in the integration chapter we meet the normed space of (equivalence classes of) Riemann integrable functions and the Banach sequence spaces l_1 and l_∞ appear in the chapter on series. This also gives the author the opportunity to introduce the concept of duality and prove that $l_1^* = l^\infty$. Another welcome guest in this last chapter is the Weierstrass approximation theorem which enables the uniform approximation of functions in $C(a, b)$ by polynomials.

Part II consists of two chapters on stand-alone topics in single-variable analysis. The first of these is an introduction to Fourier series and gives the author an opportunity to discuss Hilbert spaces via the sequence space l_2. The second is the Riemann-Stieltjes integral which leads to a proof of the Riesz representation theorem in the form that the dual space to $C(a, b)$ is the Banach space of functions of bounded variation on (a, b) equipped with the total variation norm.

In Part III, the author deals with analysis on Euclidean space. The first of four chapters sets the scene and introduces the useful topological concepts of open and closed sets, compactness and connectedness within this linear context. The next three chapters deal with continuity, differentiation (including the multivariate Taylor theorem and the inverse and implicit function theorems) and Riemann integration. The final chapter includes an introduction to both Lebesgue and Jordan null sets so that Lebesgue's criterion for Riemann integrability can be formulated and proved and also includes a version of the well-known Fubini theorem (at the Riemann integral level) which justifies the reduction of multiple integrals to iterated lower order integrals.

This is a well-written and well-structured book with clearly explained proofs and a good supply of exercises, some of which are quite challenging. Hints and solutions to a selection of these are included at the end. The author clearly loves his subject. He is passionate about the importance of proof and takes great pains, especially at the beginning of the book, to advise readers how to navigate through these and develop their understanding of the rationale and the techniques. The introduction of functional analytic ideas works well and so material in this book could be used to prepare students for later courses on Banach and/or Hilbert spaces. My only criticism of this work is that it contains none of the vector analysis that one might expect to encounter in a book entitled "advanced calculus". The gradient operator makes a brief entry on the stage so that it can be used to state the multidimensional Taylor theorem, but we don't encounter div or curl, line or surface integrals, the divergence theorem etc. Of course this would have enlarged the size of the book by fifty pages or so but the justification would be bringing together all the material from this broad topic area into a single text. Perhaps this might be considered for a second edition?

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