Review of "Hilbert Space Methods and Quantum Mechanics" by Werner O.Amrein, EPFL Press (hardback), ISBN 978-1-200-6681-4, pp.395 (2009)

Quantum mechanics is the physical theory that describes the behaviour of matter in the microscopic realm and hence reigns supreme in the realm of molecules, atoms and subatomic particles. The groundbreaking work of Heisenberg, Born, Schrödinger and others in the 1920s led to the realisation that quantum theory needed a radically different mathematical formulation to Newtonian classical mechanics. The states of quantum systems are represented by unit vectors in an infinite dimensional complex Hilbert space and observables such as position, momentum and energy are realised as (not necessarily bounded) self-adjoint linear operators acting on the space. Consequently a symbiosis has developed between the needs of physics and the mathematics of Hilbert spaces (and indeed, more general techniques in functional analysis).

The first four chapters of this book give a thorough and mathematically rigorous treatment of Hilbert space theory starting from scratch and concluding with the spectral theorem for an unbounded self-adjoint operator. The treatment is written very much with the needs of physics in mind so there is an extensive treatment of position, momentum and energy operators and their spectra. There is also a very good introduction to the tricky subject of perturbation theory which involves the study of self-adjoint operators that can be written in the form $H + V$, where H is the free Hamiltonian (Laplace operator) and V is a potential (multiplication operator).

The last three chapters deal with *scattering theory*. This is a highly technical subject that aims to describe the asymptotic (in time) behaviour of quantum particles that are scattered by a force. The subject (in which the author is a leading expert) is developed to the point where a reader should be able to engage with current research literature.

This is a well written book that would be suitable for mathematically literate graduate students who are beginning research careers in mathematical physics. Parts of the book (particularly the first four chapters) could be used to supplement a final year undergraduate course in functional analysis or mathematical foundations of quantum theory. Each chapter concludes with a set of exercises of varying difficulty, sometimes with hints towards a solution. I have a gripe with the type-setter who has overly condensed the text. The author occasionally makes too many assumptions about readers' prior knowledge, for example Bessel's inequality is not introduced in this book and is used without explanation at the top of page 18. However these are minor quibbles and don't detract from my conclusion that this is a valuable contribution to the literature on the subject.

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