Some Brief Comments on "Lévy Processes and Stochastic Integrals in Banach Spaces"

David Applebaum,
Probability and Statistics Department,
University of Sheffield,
Hicks Building, Hounsfield Road,
Sheffield, England, S3 7RH

e-mail: D.Applebaum@sheffield.ac.uk

I would like to make some brief comments about the outline proof of the Lévy-Itô decomposition which appears on p.80 of the paper [2]. In particular lets focus on the sentence:

"By the Banach space version of Glivenko's theorem, $p_{Z_{A_n^c}(t)} \Rightarrow \rho(t)$ as $n \to \infty$, and hence by the Itô-Nisio theorem, $Z_{A_n^c}(t)$ converges a.s. to a random variable $Z(t)$ whose law is $\rho(t)$."

As pointed out to me by Anna Chojnowska-Michalik, this is somewhat misleading. Glivenko's theorem is stated in Chapter 1 of [1]. It says that pointwise convergence of a sequence of characteristic functions to a limit that is itself a characteristic function implies weak convergence of the corresponding probability measures. However this result is only true in finite dimensional spaces. The appropriate theorem for infinite dimensional spaces is Proposition 1.8.1 on p.19 in [6] or Theorem 2.1.9 on p.40 in [3]. This tells us that pointwise convergence of the sequence of characteristic functions together with relative weak compactness of the corresponding probability measures is enough to give us weak convergence. For the specific set of measures that we are interested in ($\{p_{Z_{A_n^c}(t)}, n \in \mathbb{N}\}$), the required relative weak compactness is established in Theorem 3.4.9 on p.143-4 of [3]. This taken together with the limiting result for the characteristic functions obtained on page 80 of [2] gives us the weak convergence of the measures.

The second point is the Itô-Nisio theorem. This is true for sequences of independent Banach space valued random variables which have symmetric laws which is not necessarily the case here. The result that we really need is that which precedes this as Theorem 3.1.6 in [3]. This tells us that for an arbitrary sequence of independent Banach space valued random variables, convergence in distribution of the sequence of partial sums is equivalent to their almost sure convergence.

Note. Although we don't specifically need the Itô-Nisio theorem here it is worth pointing out that it has been partially generalised to arbitrary sequences of independent Banach space valued random variables (see [4], [5]).

References

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