Review of
Fred Espen Benth "Option Theory with Stochastic Analysis - An Introduction to Mathematical Finance ", Springer-Verlag Berlin Heidelberg (2004), 162 pages, £30.50 (paperback), ISBN 3-540-40502-X

The mathematical finance bandwagon continues to roll! This is the the third book on the subject that I've reviewed for the Gazette in the last year and publishers' mailshots indicate that there are many more rolling off the presses. This is a very short book (only 119 pages of text) concerned solely with describing the mathematics of option pricing and I found it a delight to read. It is very well written, quite comprehensive and non-rigorous so that it can be used on courses aimed at a variety of students.

Option pricing is essentially the business of finding fair prices of financial derivatives, i.e. contracts which confer on the holder the right, but not the obligation to buy shares on (or perhaps before) a given time. Due to ground breaking work by F.Black, M.Scholes and R.Merton during the 1970s, we now know that the prices of many different types of options have explicit stochastic representations as expectations of certain random processes. The key to understanding this is the stochastic calculus originally developed by the Japanese probabilist K.Itô in the 1940s. Itô found a way of constructing integrals of suitable random processes against "bad integrators" such as Brownian motion, which cannot be dealt with by the standard Riemann or Lebesgue formalisms. The study of the resulting "stochastic integrals" and associated "stochastic differential equations" leads to a new type of calculus based on infinitesimal random changes. Since Itô's groundbreaking work there have been a wealth of both theoretical and applied developments in the "stochastic calculus", not least in relation to mathematical finance.

In this book, the author gives a self-contained introduction to stochastic calculus, indeed he even includes an account of all the basic probability which is needed. He then tackles the option pricing problem and shows how it can be solved - first as the solution of the "Black-Scholes partial differential equation" and then describing this solution using the expectation with respect to a "risk neutral measure", i.e. the unique probability measure which makes the discounted stock price a martingale. This is then applied to give the explicit "Black-Scholes formula" for the price of a European call option in terms of the cumulative distribution function for the standard normal. He also explains the important concept of "dynamic hedging" whereby the price of the option at any time is reproduced by the value of a portfolio comprising stocks and bonds alone.

The account of option pricing given above is now standard and can be found in many textbooks. It is however based on a model whereby stock prices are driven by Brownian motion. This has the theoretical advantage that the market is "complete" (i.e. a hedging strategy exists for a large class of options) and the pedagogic advantage that much can be computed via the normal distribution. However there is a significant amount of empirical evidence to suggest that this assumption is not justified and there has been a great deal of research in recent years on alternative models. It is nice to see this problem being tackled head-on in a book of this type - indeed the author devotes an early chapter to it and gives an introduction to the normal inverse Gaussian model which is one of the candidate processes to supplant Brownian motion.

In a final chapter, the author discusses numerical approaches - specifically the finite difference method for the Black-Scholes pde and the Monte Carlo method for the probabilistic representation of its solution. Some programmes to implement the algorithms are presented using Visual Basic. The book includes a healthy number of exercises and there are fully worked solutions to most of these.

It is difficult to find fault with such a well written book - but if a second edition is planned, I have a couple of suggestions to make. First of all, many students coming to option pricing for the first time find it easier to understand continuous time pricing if they have first of all encountered the more accessible discrete time model. At present the author rushes through this in four pages and only deals with the one-period market. It would have been useful to have included a full account of the multi-period binomial model. Secondly, the book treats European options in some detail and says something about more exotic options such as Asian and knock-out options, however the majority of those traded in real markets are American options. These are more difficult to model mathematically as they involve an optional stopping problem but at least some brief account of this would be welcome.

Finally, for the publishers to price such a short book at the exorbitant sum of £30.50 strikes this reviewer as crazy !

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