Review of "Markov Processes, Brownian Motion and Time Symmetry" by Kai Lai Chung and John B.Walsh (second edition), Springer Verlag (2005). 444 pp, price £84.50, ISBN:978-0-387-22026-0

In 1687 Isaac Newton completed the third volume of his monumental "Principia" in which he postulated the celebrated inverse square law for the gravitational force between two massive bodies. Nearly one hundred years later in 1785 Charles Augustin de Coulomb found a similar law to hold for the force between two magnets and he later extended this to electric charges. In all three cases the *potential* energy is inversely proportional to the distance between the objects in question. Generalising the three dimensional case, we find that if $d \geq 3$ and $|x|$ is the Euclidean distance between a vector x in $\mathbb{R}^d$ and the origin then the function $\psi(x) = |x|^{2-d}$ is harmonic, i.e. it satisfies Laplace's equation $\Delta\psi = 0$. *Potential theory* is the deeper ramification of these ideas e.g. the classical Dirichlet problem seeks to uniquely construct a harmonic function ψ in a domain D from the knowledge that ψ coincides precisely with a given smooth function f on the boundary ∂D.

In the twentieth century these ideas were given a probabilistic twist through the works of Shizuo Kakutani, Joseph Doob and many others. For example it was shown that the classical Dirichlet problem can be solved by running a Brownian motion in the region D. More precisely if $(B_x(t), t \geq 0)$ is Brownian motion conditioned to start at $B_x(0) = x$ and τ_D is the first time that this process hits the boundary ∂D, then the required harmonic function is $\psi(x) = \mathbb{E}(f(B_x(\tau_D)))$ where $\mathbb{E}$ denotes expectation. Thus was born *probabilistic potential theory* which is the subject of this book.

In fact the volume under review is the union of two distinct albeit complementary works. The cover suggests that it is a "second edition" but this isn't quite the whole story. The first half of the book comprises a genuine second edition of the classic text "Lectures from Markov processes to Brownian motion" by the first named author which was originally published in 1982. The second half of the book is a new contribution which is solely due to the second named author.

Probabilistic potential theory is a highly technical and complex subject. Roughly speaking it aims to extend and develop the potential theoretic aspects of Brownian motion (as indicated above) to more general classes of Markov processes - particularly Feller and Hunt processes (see Chung's part of the book) and Ray processes (which can be found in Walsh's part.) Nonetheless there are a number of interesting examples in the more familiar world of Markov chains which the reader can use to gain insight into the new ideas. The mathematical development relies on the modern theory of stochastic processes and semigroup techniques (in the functional analytic sense) are essential. Nonetheless the hallmark of the subject is the influence of physical ideas as is indicated by the names of key concepts such as equilibrium measure and capacity.

If I were asked to recommend a book to a beginner who wished to immerse themselves in the subject and emerge in a fit state to tackle the contemporary literature, then I would choose this volume without hesitation. It introduces important concepts with great care and clarity, there are a healthy set of exercises to practice on and each chapter closes with interesting historical comments and pointers to related literature. However you shouldn't even pick this book up unless you have first mastered a graduate level course in measure theoretic probability and learned something along the way about Brownian motion, martingales and Markov processes. For a gentler introduction to the subject, readers might consult K.L.Chung "Green, Brown and Probability and Brownian Motion on the Line", World Scientific (2002).

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