Authors' comments on "Probabilistic Approach to Fractional Integrals and the Hardy-Littlewood-Sobolev Inequality"

In the statement of the Sobolev inequality using Dirichlet forms (7 lines down on p.6), the left hand side should be $||f||^2_{\frac{2n}{n-2}}$.

In calculations using (2.1) we have often implicitly taken $D_p = 1$. This does not affect any of the proofs, for if D_p had been explicit, it would only have been absorbed into a generic constant.

In the proof of Lemma 4.2, 5 lines from the bottom of page 14, change $0 \le t \le a$ to $0 \le t < a$. Alternatively, to show the square-integrability one can proceed as follows.

$$\begin{aligned}
\int_0^t \mathbb{E}|\nabla T_{2(a-s)} f(B_s)|^2) ds &\le \int_0^a \mathbb{E}|\nabla T_{2(a-s)} f(B_s)|^2 ds \\
&= \int_0^a \int_{\mathbb{R}^d} |\nabla T_{2(a-s)} f(x)|^2 dx ds \\
&= \int_0^a \int_{\mathbb{R}^d} |\nabla T_{2s} f(x)|^2 dx ds \\
&\le \int_0^\infty \int_{\mathbb{R}^d} |\nabla T_{2s} f(x)|^2 dx ds \\
&= \int_0^\infty \int_{\mathbb{R}^d} e^{-2s|\xi|^2} |\xi|^2 |\widehat{f}(\xi)|^2 ds d\xi \\
&= \frac{1}{2} \int_{\mathbb{R}^d} |\widehat{f}(\xi)|^2 d\xi = \frac{1}{2} \int_{\mathbb{R}^d} |f(x)|^2 dx
\end{aligned}$$

Integration by parts could also be used in place of the Fourier transform and Plancherel.