Review of
Kyosi Itô "Stochastic Processes: Lectures Given at Aarhus University", ed. O.E.Barndorff-Nielsen and K.-I.Sato, Springer-Verlag Berlin Heidelberg (2004), 234 pages, £46 (hardback), ISBN 3-540-20482-2

I'll begin with a health warning. Most books with a similar title to this are introductory accounts of the subject dealing with standard topics such as Markov chains, the Poisson process etc. The book under review is in fact an advanced text suitable for graduate students and based around two topics - the structure of additive processes (stochastic processes with independent increments) and the basic theory of Markov processes, which generalises Markov chains to continuous time and fairly general state spaces.

The author K.Itô is one of the most distinguished probabilists of the twentieth century. Among his many achievements was the creation of the stochastic calculus, a subject which continues to undergo intensive development and which has a wealth of applications, most notably to mathematical finance. The lectures on which this book is based were delivered at Aarhus University in 1968-9 and the mimeographed version of these have been popular for many years. Part of the rationale for making this material more widely available in book form must be the first part where the so-called Lévy-Itô decomposition is given a thorough rigorous proof. This fundamental result was first postulated by the great French probabilist Paul Lévy and then made rigorous by Itô himself in a paper published in 1942. The basic idea is that any additive process may be split into the sum of three independent additive processes - these being a path continuous process which has a Gaussian distribution, a sum of "large jumps" and a more complicated term which corresponds to "small jumps". Both of the latter two terms can be described using integration with respect to a Poisson random measure. In recent models of stock price dynamics in mathematical finance, the small jumps correspond to the usual jitter whereby prices dance randomly within a narrow range while the large jumps correspond to major shocks which might, for example, follow a major earthquake or a terrorist atrocity. The proof given here by Itô is along classical lines. An alternative approach making greater use of martingales can be found in some other recently published monographs.

The second half of the book is a nice introduction to Markov processes making extensive use of semigroup techniques. An explicit path to potential theory is traced using the canonical example of Brownian motion. The book concludes with a number of exercises accompanied by worked solutions.

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