

**Corrections to “Lévy Processes and Stochastic
Calculus” - by D.Applebaum December 2004/January
2005**

*These corrections are all incorporated into the second printing -
available in summer 2005.*

1. General Problem. From [250] onwards there is a problem with the referencing. Patterson[250] is OK, but Picard is also [250] in the text (but not in the index) and the problem remains from then on.

2. Local points

(xii), -2 “its” should be “it’s”.

17, +4 Change $\tilde{\psi} = \psi + ix$ to $\tilde{\psi}(\cdot) = \psi(\cdot) + ix$

91, equation (2.6) Replace $\sum_{n \in \mathbb{N}} f(\Delta(X(T_n^A \wedge t)))$ with $\sum_{n \in \mathbb{N}} f(\Delta(X(T_n^A))) \chi_{[0,t]}(T_n^A)$

96, -3 should read

$$\mathbb{E}[(M_1(t), M_2(t))] = \mathbb{E} \left(\sum_{0 \leq s \leq t} (\Delta M_1(s), \Delta M_2(s)) \right).$$

p.96, -2 After “Proof”, insert “For convenience, we work in the case $d = 1$ ”.

p.104, + 7 Change X to x .

Equation (2.15) should read

$$\mathbb{E}(|M(t, A_n)|^2) = \sum_{m=1}^n \mathbb{E}(|M(t, B_m)|^2)$$

Lines -12 to -10 should read

$$\text{Var}(|\hat{Y}(t)|) = \text{Var}(|\hat{Y}(t) - M(t, A_n)|) + \text{Var}(|M(t, A_n)|).$$

Hence $\mathbb{E}(|M(t, A_n)|^2) = \text{Var}(|M(t, A_n)|) \leq \text{Var}(|\hat{Y}(t)|) \dots 2.16$

Equation on line -6 should read

$$\mathbb{E}(|M(t, A_{n_2}) - M(t, A_{n_1})|^2) = \mathbb{E}(|M(t, A_{n_2})|^2) - \mathbb{E}(|M(t, A_{n_1})|^2).$$

page 105, lines -13 to -14 should read

$$\begin{aligned} 0 &\neq \mathbb{E} \left(\left(Y_c(t), \int_{|x|>b} f(x) \tilde{N}(t, dx) \right) \right) \\ &= \lim_{n \rightarrow \infty} \mathbb{E} \left(\left(\hat{Y}(t) - M(t, A_n), \int_{|x|>b} f(x) \tilde{N}(t, dx) \right) \right) = 0, \end{aligned}$$

p.118, +4 Replace $\Delta f(t)$ with $|\Delta f(t)|$.

121, +10 Delete “on $B_b(\mathbb{R}^d)$ ” and replace with “from $B_b(\mathbb{R}^d)$ to the Banach space (under the supremum norm) of all bounded functions on \mathbb{R}^d .”

121, +13 After “ $B_b(\mathbb{R}^d)$.”, insert “We say that the Markov process X is *normal* if $T_{s,t}(B_b(\mathbb{R}^d)) \subseteq B_b(\mathbb{R}^d)$, for each $0 \leq s \leq t < \infty$.”

121, +14 After ”Theorem 3.1.2” insert “If X is a normal Markov process, then”

122, +13 Delete “By” and replace with “If X is an arbitrary Markov process, by ”

122, +16 Start new line before Exercise 3.1.3 with “From (3.3) we see that a Markov process is normal if and only

if the mappings $x \rightarrow p_{s,t}(x, A)$ are measurable for each $A \in \mathcal{B}(\mathbb{R}^d)$, $0 \leq s \leq t < \infty$.”

Normal Markov processes are a natural class to deal with from both analytic and probabilistic perspectives, and from now on we will concentrate mainly on these.

p.123 Delete lines 1-5.

page 126, -13. Between “below.” and “For more on...”, insert “ In particular, for most of semigroups which we study in this book, condition (2) above fails when we replace $C_0(\mathbb{R}^d)$ with $C_b(\mathbb{R}^d)$.”

p.185, -8 to -9 , Replace “ $\hat{f} \in L^1(\mathbb{R}^d, \mathbb{C})$, where” by “ \hat{f} , defined by”

p.185, -6 to -7, Replace “ \mathcal{F} is a bounded linear operator on $L^1(\mathbb{R}^d, \mathbb{C})$ ” with “ \mathcal{F} is a linear mapping from $L^1(\mathbb{R}^d, \mathbb{C})$ to the space of all continuous complex valued functions on \mathbb{R}^d .”

p.223, equation (4.11) In the integral on the RHS, change $K^i(t, x)$ and $N(dt, dx)$ to $K^i(s, x)$ and $N(ds, dx)$, respectively.

p.258, line 1, change “local martingale” to “local P -martingale”.

line 4, change “local martingale” to “local Q -martingale”.

line 5, change full stop after integral to a comma and insert “and we are assuming that the integral exists. A sufficient condition for this is that $\int_0^t \int_{|x|<1} |e^{H(s,x)} - 1|^2 \nu(dx) ds < \infty$.”

p.265, -7 insert “a unique” after “there exists”

-1, Delete “and the result follows.”, insert “Uniqueness follows from the easily proved fact that the Itô isometry is injective.”

-1 Change (a.s.) to (a.e.)

p.300,+6 Insert “,where $k > 2$, after $\mathcal{L}_k(\mathbb{R}^d)$ (also deleting the full stop).

p.338, + 1 Change 58-62 to 158-162

p.338, +2,+3 and 339, -5, Change $C(p)$ to $C(p, t)$.

p.340, +8 Delete “If” which follows **Theorem 6.74**.

p.341, +5 After “as $t \rightarrow 0$.”, insert “The fact that $\lim_{t \rightarrow 0} \|T_t f - f\| \rightarrow 0$, for all $f \in C_0(\mathbb{R}^d)$ follows by a straightforward density argument.”

p.353, Delete lines -4 to -8 and replace with “If c is such that $\mathcal{N} : C_0^2(\mathbb{R}^d) \rightarrow C_0(\mathbb{R}^d)$ then $(T_t, t \geq 0)$ is a Feller semigroup by theorem 6.7.4. The following exercise gives a sufficient condition on c for this to hold. ”

p.353, -2 to -1, Replace “ $C_b(\mathbb{R}^d)$ as above and also $\dots = 0$ ” with “ $C_0(\mathbb{R}^d)$ ”.

p.354, +1, Replace $[\xi(x)(y) - y]$ with $|\xi^i(x)(y) - y^i|$.