Review of "Limit Theorems for Stochastic Processes" (second edition), by Jean Jacod and Albert N. Shiryaev

661 pages, hardback £84, ISBN 3-540-43932-3 Springer-Verlag Berlin Heidelberg New York (2003)

This book concerns the interaction of two of the most important themes in modern probability theory - limit theorems and stochastic processes which describe random "noise". Limit theorems are a classical theme in probability theory, which are vital for understanding how a myriad of microscopic random influences can combine together to produce a macroscopic effect. The most well-known of these results is the celebrated central limit theorem which can be traced back to work of de Moivre and Laplace. In its modern form, this states that if $(X_n, n \in \mathbb{N})$ is a sequence of independent, identically distributed random variables with common mean b and standard deviation σ, then $\frac{X_1 + X_2 + \cdots + X_n - nb}{\sqrt{n}\sigma}$ converges in distribution as $n \to \infty$ to a standard normal. At the level of stochastic processes, Donsker proved that (taking $b = 0$ and $\sigma = 1$), then $(X_n(t), t \geq 0)$ converges in law as $n \to \infty$ to standard Brownian motion, where each $X_n(t) = \frac{1}{\sqrt{n}} \sum_{1 \leq j \leq [nt]} X_j$ (with $[\cdot]$ denoting 'integer part of').

Brownian motion is the best-known of those processes which can be used to describe random "noise". It was developed as a model for the corresponding physical phenomenon by Einstein and Smoluchowski, and as a description of stock price movements by Bachelier, in the early years of the twentieth century. Brownian motion was first made mathematically rigorous by Norbert Wiener in the 1920s and this work has led to a wealth of important theoretical and applied developments. One of the most significant achievements, for both of these was the creation of stochastic calculus by Kyosi Itô in the 1940s. This involved the development of a stochastic integral with Brownian motion as integrator and hence to the construction of stochastic differential equations, which infinitesimally describe motion having both deterministic and random components. In recent years, these ideas have found spectacular application to financial mathematics, particularly the option pricing formula due to Black, Scholes and Merton. The latter (surviving) two of these received the 1997 Nobel Prize for Economics in recognition of this achievement.

During the 1970s, it became clear that the natural setting for stochastic calculus was the class of processes called 'semimartingales'. Technically, these are sums of local martingales (processes which might model a 'fair game') and a process of finite variation. They include many obvious candidates which might model noise including Brownian motion, the Poisson process and processes with stationary and independent increments. It is only in very recent years, that probability theorists have begun to go beyond the semimartingale paradigm and consider noise processes such as fractional Brownian motion which have nice self-similarity properties, but are no longer semimartingales.

The main theme of this book is to use the generality of the semimartingale concept to give a unified treatment of a large class of limit theorems. The latter range from classical results on the convergence of asymptotically negligible, triangular arrays of random vectors to more modern results on the convergence of sequences of Markov and diffusion processes. The first edition of this work was published in 1987 and it has long been recognised as a classic within the probability community. The new edition is very welcome. Changes have been introduced in a minimal way, usually as additions at the end of each chapter. Much of the new material has been included because of its importance for financial applications.

Prospective readers should be warned that this is a very advanced book, designed for use by graduate students and research workers in probability. It contains a wealth of valuable material, not just on limit theorems, but on the basic structure of semimartingales, the general theory of processes and the construction and properties of stochastic integrals. For those with an interest in statistics, there is also a thorough treatment of contiguity and of limit theorems for sequences of likelihood ratios.

The scope and depth of this important treatise are remarkable. No university library which claims to cater for research in probability can afford to be without it. I have no doubt that the new edition will be a vital resource for many future generations of probability theorists.

David Applebaum

(The Nottingham Trent University)